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Detecting the relative localisation of quantum particles

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Abstract. One interpretation of how the classical world emerges from quantum physics involves the build-up of certain robust entangled states between particles due to scattering events [1]. This is intriguing because it links classical behaviour with the uniquely quantum effect of entanglement and differs from other interpretations that say classicality arises when quantum correlations are lost or neglected in measurements. However, the problem with this new interpretation has been finding an experimental way of verifying it. Here we outline a straightforward scheme that enables just that and should, in principle, allow experiments to confirm the theory to any desired degree of accuracy.

1. Introduction

A lot of work has been done to understand why apparently different physical theories should apply to microscopic and macroscopic systems and how one theory crosses over to the other in the mesoscopic regime. The prevailing view is that the emergence of classical physics from a more fundamental quantum reality can be interpreted in terms of decoherence [2, 3, 4, 5]. Simply put, this says that quantum systems tend to interact with their environments and become entangled with them. The total system including the environment is therefore properly treated with quantum physics. However if we are interested only in the quantum subsystem we neglect the information about which environmental states are correlated with which subsystem states. We then find that the subsystem appears to behave more and more classically the more it has interacted with the environment. In effect, by throwing away information about the quantum correlations we are left with a system that behaves classically.

Another interpretation that extends this idea was put forward a few years ago [1]. It showed the emergence of classicality without having to throw away all the quantum correlations. In fact it showed that even classical objects are entangled with one another but with a special type of robust entanglement sometimes called “fluffy-bunny” entanglement [6, 7]. This is an appealing view since it gives one consistent theory that describes both quantum and classical systems. It is also intriguing that in this theory classicality is associated with entanglement, which is usually thought of as a purely quantum feature. Furthermore, this interpretation gives a clear basis to the idea that we should only think in terms of relative (rather than absolute) positions in physics.

The problem with this formalism up until now is that it has not been at all clear how it could be tested experimentally. In this paper we resolve this issue by providing a simple,



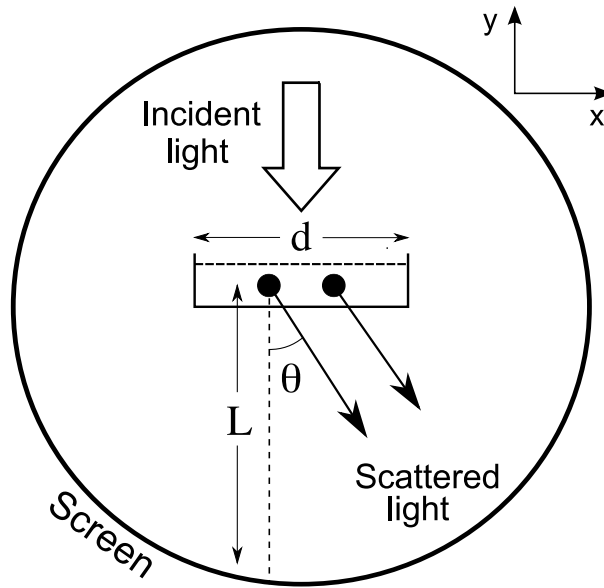


Figure 1. A schematic showing the setup. Two massive particles are delocalised over some region d and are illuminated by plane wave incident light. The scattered light is detected at an angle θ on a screen located at a distance L away from the particles. For clarity, the diagram is not to scale: we consider the case where $L \gg d$.

experimentally accessible scheme that could clearly demonstrate this process. In principle, it should allow experiments to confirm to any level of certainty that this process has occurred.

2. The scheme

We begin by reviewing the scheme [1] for measurement-induced relative-position localisation through entanglement. The setup is shown in Fig. 1. Two distinguishable massive particles are delocalised over some region d in the x -direction in the sense that their de Broglie wavelengths are comparable to d in this dimension. To simplify the following analysis we will take these particles to be tightly confined in the y and z directions, although this assumption is not necessary. These two particles will form the sub-systems of the system we are interested in. They are illuminated with plane-wave light with wavelength λ incident along the y -axis, which scatters from them and is detected at an angle θ on a screen located at a distance L away. We will consider the far-field limit where $L \gg d$.

The initial wave function of the particles in the relative position representation can be written as

$$|\psi\rangle = \int_{-d}^d c(x)|x\rangle dx, \quad (1)$$

where $|x\rangle$ represents the *relative* position of the two particles in the x -direction, $c(x)$ is the probability amplitude for x and $\int_{-d}^d |c(x)|^2 dx = 1$. Since the particles are delocalised over d , the position of one particle relative to the other lies in the range $[-d, d]$. Strictly, to give a full spatial description of the system we should specify the centre-of-mass position as well as the relative position. However the centre-of-mass remains unentangled from the relative position coordinate throughout the scattering process and so can be conveniently neglected. When a photon of wavelength λ scatters off a particle into direction θ , its momentum in the x -direction changes by $\Delta p = h \sin \theta / \lambda$, where h is Planck's constant. The particle from which it scatters

therefore gets an equal and opposite kick. In relative momentum space the particles therefore receive a kick of $\pm h \sin \theta / \lambda$ depending on which particle the photon scatters from and, since we do not know, we get a superposition of both possibilities.

This allows us to write the overall state of the system after a photon has scattered as

$$|\psi\rangle = \int_{-d}^d dx c(x)|x\rangle \otimes \left[\frac{1}{2\sqrt{2\pi}} \int_0^{2\pi} d\theta \left(e^{i2\pi x \sin \theta / \lambda} + e^{-i2\pi x \sin \theta / \lambda} \right) |\theta\rangle + A(x)|0\rangle \right]. \quad (2)$$

The first part of the tensor product is the state of the two particles in relative position space and the second part (in square brackets) is the state of the scattered photon where $|\theta\rangle$ represents a photon scattered at angle θ . The term proportional to $A(x)$, defined as

$$A(x) = \left[\frac{1}{2\pi} \int_0^{2\pi} \sin^2(2\pi x \sin \theta' / \lambda) d\theta' \right]^{1/2}, \quad (3)$$

represents a nonscattering event that leaves the photon in the undeflected state $|0\rangle$. This term is necessary because the total rate of scattering (integrated over all angles) depends on the separation of the particles, x . Odd as it seems at first sight, this means that detecting a photon that is not scattered gives us information about the relative position of the particles. The $A(x)$ term is required to properly account for this.

We can now understand the localisation process. The probability density for detecting a scattered photon at angle $\theta \neq 0$ is

$$P_S(\theta) = \langle \psi | a_\theta^\dagger a_\theta | \psi \rangle = \frac{1}{2\pi} \int_{-d}^d |c(x)|^2 \cos^2(2\pi x \sin \theta / \lambda) dx,$$

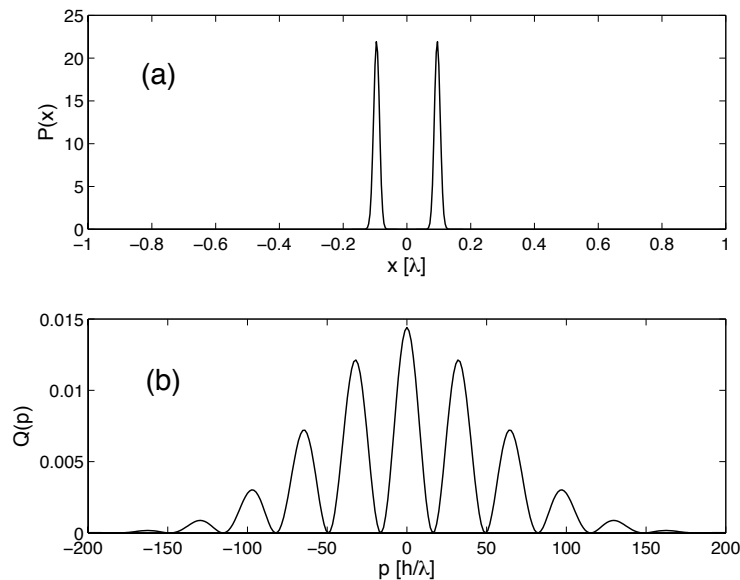


Figure 2. The case of light scattering causing relative localisation. (a) Probability density, $P(x)$, for the relative position of the particles after the scattering and detection of 150 photons. The position is given in units of the wavelength, λ , of the scattered light. (b) Probability density, $Q(p)$ for the corresponding relative momentum of the particles.

where $|\psi\rangle$ is given by Eq. (2) and a_θ is the annihilation operator for the photonic mode $|\theta\rangle$. The probability of detecting a nonscattered photon, i.e. $\theta = 0$, is $P_{NS} = \int_{-d}^d |c(x)|^2 A^2 dx = 1 - \int_0^{2\pi} P_s(\theta) d\theta$.

To simulate the localisation process, we draw a random number from this distribution to see whether the photon is scattered and, if so, at what angle. If it is not scattered the (unnormalised) new state is $a_0|\psi\rangle = \int_{-d}^d dx c(x) A(x)|x\rangle$ and if it is scattered at an angle θ_1 , the (unnormalised) new state is $a_{\theta_1}|\psi\rangle = \int_{-d}^d dx c(x) \cos(2\pi x \sin \theta_1/\lambda)|x\rangle$. We then normalise the state and repeat for the next photon.

3. Results

We choose to start our simulations with a flat distribution, $c(x) = 1/\sqrt{2d}$ because we want to choose the ‘hardest’ case and show that relative localisation builds up even when there is none to begin with. This choice does not restrict the generality of the results and qualitatively similar outcomes are obtained for different choices. The probability distribution, $P(x)$, for the relative position of the two particles is shown in Fig. 2(a) for a typical run after 150 photons have been detected and for $d = \lambda$. We assume that the 150 photons are all incident on the particles in a sufficiently short time period that we do not need to consider the dynamics of the particles between detection events. Initially the distribution is completely flat and we see that the measurement process has induced localisation. The distribution shown in Fig. 2(a) after 150 photons have been detected has two peaks that are symmetric about the origin. This is what we would expect since the detection process cannot determine which particle is on the left and which is on the right and so the relative position must have a positive and negative component. If the two particles had a well-defined relative position to begin with, then there would be only one peak in this distribution¹ as shown, for example, in Fig. 3(a).

The relative-position localisation process is analogous to the build-up of relative phase between two number state Bose-Einstein condensates when interference patterns are detected between them [8, 9, 10]. Just like in that case we cannot distinguish from the detected particles whether the position (or phase of the BECs) was well-defined to begin with or created by the measurements. We need a way of doing this in order to experimentally verify that the localisation process takes place. Although the distinction between Fig. 2(a) and Fig. 3(a) is clear, an experimentalist would not have direct access to this since the detected photons cannot tell these distributions apart. One possible solution is to look in the conjugate space – in this case relative momentum. A similar idea has been applied to BECs [11, 12, 13].

The relative momentum distributions corresponding to the relative position distributions in Figs. 2(a) and 3(a) are shown as the solid lines in Figs. 2(b) and 3(b) respectively. For ease of comparison, the result from Fig. 2(b) is superimposed on Fig. 3(b) as a dashed line. We see that the two distributions have the same envelope, but the case where localisation is induced has interference fringes. For particles that are *a priori* perfectly localised, the distribution in Fig. 3(a) would be a delta function and the momentum distribution would be completely flat. We have chosen the relative position distribution shown because it is an upper limit to the width possible based on the photons detected. In other words, it is the ‘hardest’ case to distinguish from that shown in Fig. 2(a). We want to demonstrate that our technique works even in this worst-case scenario.

The measurement scheme proceeds as follows. After scattering the photons from the particles,

¹ Strictly, in the case where the particles are each initially localized, we have a mixture of the two different relative positions. This mixture reflects the classical uncertainty in our knowledge of the relative position of the particles based on detecting the scattered photons. This does not change the shape of Q_2 . The case where the particles are initially delocalized is quite different and gives a coherent superposition of the two relative positions, which leads to the distribution Q_1 .

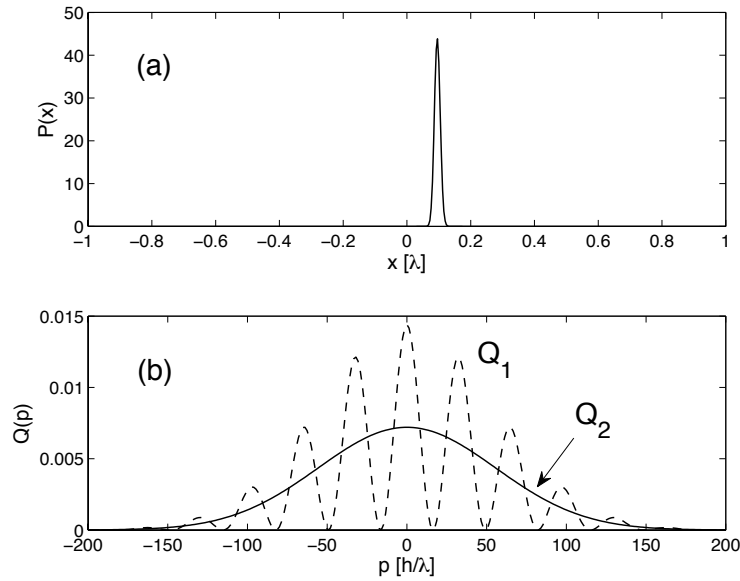


Figure 3. The same as in Figure 2 but with the particles initially localised before the photons are scattered. (a) There is now only one peak in the relative position or, strictly, an equally weighted mixture of the two peaks, both of which give the same relative momentum distribution. (b) The corresponding relative momentum probability density is shown as a solid line (labelled Q_2) and is compared to the result in Figure 2 shown as a dashed line (labelled Q_1).

we want to distinguish the two relative momentum distributions shown in Fig. 3(b). To do this, we switch off any trapping potential and allow the particles to move freely. By detecting their positions in the x -direction after some time of flight, we can infer the x -components of their momenta and hence the relative momentum of the particles in that direction. By repeating the whole process from the beginning many times, we should be able to build up a probability distribution and so distinguish the two cases. However the stochastic nature of the process means that the particles localise to a different relative position on each run and so the relative momentum fringes are different each time. If we were to just naïvely add the results from each run, the fringes would wash out. Instead we can use Bayesian analysis to precisely distinguish the two scenarios.

Suppose on a particular run we detected scattered photons on the screen that meant the relative momentum distribution was either $Q_1(p)$ or $Q_2(p)$ depending on whether or not the scattering process induced relative localisation (see Fig. 3(b)). To begin with, we do not know whether the particles are localised or not so we take our prior probability of them initially being localised, P_l to be the same as the prior probability of them not initially being localised, P_{nl} , i.e. $P_l = P_{nl} = 0.5$. Now suppose, upon releasing the particles, we measure their relative momentum to be p_1 . This gives us some information about which scenario is more likely. In particular, Bayes theorem tells us that the updated probabilities are $P_{nl} \propto Q_1(p_1) \times 0.5$ and $P_l \propto Q_2(p_1) \times 0.5$. Normalising, we get

$$P_{nl} = \frac{Q_1(p_1)}{Q_1(p_1) + Q_2(p_1)} \times 0.5 \quad (4)$$

and $P_l = 1 - P_{nl}$. We can then iterate this process by using these updated probabilities as the prior probabilities in the next step. By repeating many times we increasingly refine our

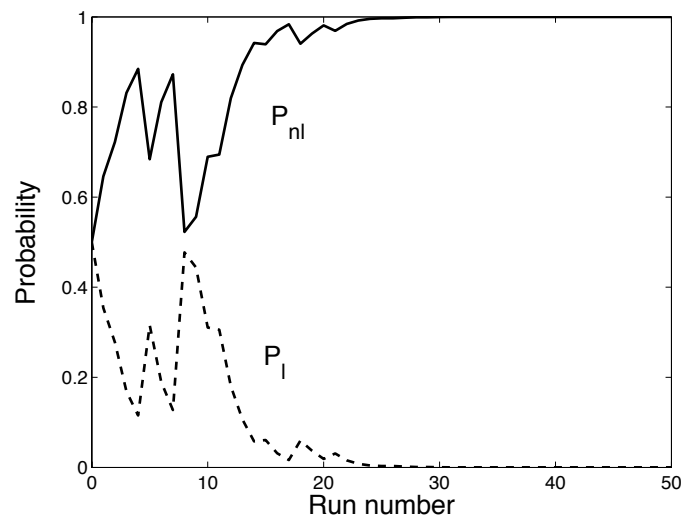


Figure 4. A simulated experiment showing the Bayesian analysis of the probability P_{nl} , that the photon scatterings caused relative localisation of the particles (solid line) and the probability, P_l , that they were localised to begin with (dashed line). In our simulation, we have taken the particles to start off with no well-defined relative position.

knowledge of which process is occurring.

A sample simulation is shown in Fig. 4 for the case that the particles do not initially have a well-defined relative position. We see that initially $P_{nl} = P_l = 0.5$ and that as more and more runs are performed our knowledge of what process is occurring is refined. The probabilities initially jump around for a while before settling down after about 25 runs. The information in this figure is what would be directly accessible to experimentalists and so, in this case, they would be quite certain after about 25 runs that they had observed measurement-induced relative-position localisation. The experimentalists can stop the experiment as soon as the desired level is reached.

4. Conclusion

We have presented a conceptually simple scheme that should enable experiments to detect relative position localisation for quantum particles. This detection scheme has been the missing element in an interpretation for how scattering events could lead to the emergence of classical-like behaviour in quantum systems. This idea could have important consequences for our understanding of the boundary between quantum and classical physics and the role of relative observables in nature.

Acknowledgments

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